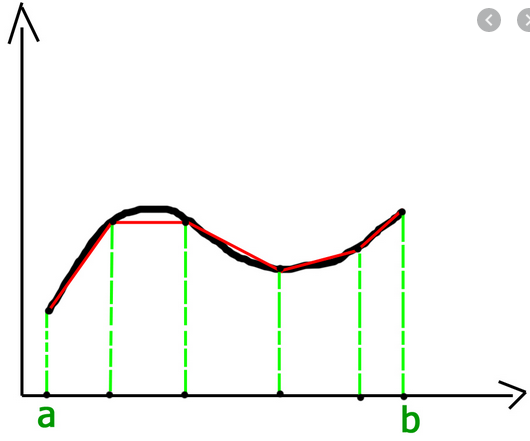
***Definite Integration***

Let  be a function which is continuous on the closed interval . The definite integral of  from  and is defined to be the limit



where  is a Riemann sum of  on . So a definite integral is an integral  with upper and lower limits. If  is restricted to lie on the real line, the definite integral is known as a Riemann integral. However, a general definite integral is taken in the complex plane, resulting in the contour integral with  and  in general being complex numbers and the path of integration from  to  known as a contour.

**Integration as the limit of a sum:** Let, be a continuous, bounded and single-valued function defined in the interval where *a*, *b* are finite quantities and.



If the interval be divided into *n* equal sub-intervals, each of length *,* by the points  so that , then the area enclosed by is defined as

S =





 where  if then .

Which is also defined as the definite integral of with respect to *x* between the limits *a* and *b*, and is denoted by the symbol,



where, *a* is called the lower limit and *b* is called the upper limit.

Therefore,  where .

***NOTE:***

1. 
2. 
3. .

**Problem-01:** Evaluate  from the definition of the integral as the limit of a sum.

**Solution:** We have 

Here 



Since 

where 











.

**Problem-02:** Evaluate  from the definition of the integral as the limit of a sum.

**Solution:** We have 

Here 



Since 

where 



























**Problem-03:** Evaluate  from the definition of the integral as the limit of a sum.

**Solution:** We have 

Here 



Since 

where 











.

NOTE: .

**Problem-04:** Evaluate  from the definition of the integral as the limit of a sum.

**Solution:** We have 

Here 



Since 

where 













NOTE: 

**Problem-05:** Evaluate  from the definition of the integral as the limit of a sum.

**Solution:** We have 

Here 



Since 

where 



; 

















There is another definition of a finite integral as the limit of a sum and is generally used for evaluating by summation , where *m* is a positive integer or a negative integer or a positive or negative fraction.

If  is continuous and single valued in the closed interval , then



where .

**Problem-06:** Evaluate  from the definition of the integral as the limit of a sum.

**Solution:** We have 

Here ,  and .



Since 

where, 



















**Problem-07:** Evaluate

**Solution:** Given that,















**Problem-08:** Evaluate

**Solution:** Given that,

















**Problem-09:** Evaluate

**Solution:** Given that,

















**Problem-10:** Evaluate

**Solution:** Given that,

















**Problem-11:** Evaluate

**Solution:** Given that,



















**Problem-12:** Evaluate

**Solution:** Given that,























**Problem-13:** Evaluate

**Solution:** Given that,

















**Problem-14:** Evaluate

**Solution:** Given that,

















**Assignment:**

**Problem-01:** Evaluate

**Problem-02:** Evaluate

**Problem-03:** Evaluate  from the definition of the integral as the limit of a sum.

**Problem-04:** Evaluate  from the definition of the integral as the limit of a sum.

**Problem-05:** Evaluate  from the definition of the integral as the limit of a sum.

**Problem-06:** Evaluate  from the definition of the integral as the limit of a sum.

**Theorem-01:** State and prove Fundamental theorem of Integral Calculus.

**OR**

State and prove the first Fundamental theorem of Calculus.

**Statement:** If be a bounded and continuous function defined in the interval where, and there exists a function  such that , then



This is called the fundamental theorem of integral calculus.

**Proof:** Let be any points in  such that



Since  and , so  and . These points divide  into subintervals , , , ,  whose lengths are denoted by .

i.e. .

Since is an anti-derivative of on  i.e  for all  on , so  satisfies the hypothesis of the mean value theorem of differential calculus on each  subintervals , , , , .

Then by the Lagrange’s Mean value theorem of differential calculus we can find points  in the respective subintervals , , , , .

i.e. ,,

such that

































Adding (1) to (*n*), we get



We now allow  i.e. the numbers of sub-intervals is infinity in such a way that  and  , then by the definition of definite integrals we have



Now taking limit on both sides of (*i*) we get





 **(Hence proved)**

***Some Definite integration***

**Problem-01: Evaluate** 

**Solution: Let,** 















**Problem-02: Evaluate** 

**Solution: Let,** 













**Problem-03: Evaluate** 

**Solution: Let,** 











**Problem-04: Evaluate** 

**Solution: Let,** 













**Problem-05: Evaluate** 

**Solution: Let,** 

























**Problem-06: Evaluate** 

**Solution: Let,** 





















**Problem-07: Evaluate** 

**Solution: Let,** 







put 

when then 

when then 

Now, 









**Problem-08: Evaluate** 

**Solution: Let,** 





put, 

when then 

when then 

Now, 











**Problem-09: Evaluate**  **Exercise-01:**

**Solution: Let,**  **Ans:**

 **Exercise-02:**

 **Ans:**





put, 

when then 

when then 

Now, 





















**Problem-10: Evaluate**  **Exercise-03:**

**Solution: Let,**  **Ans:**

 **Exercise-04:**

 **Ans:**





put, 

when then 

when then 

Now, 









**Problem-11: Evaluate** 

**Solution: Let,** 

put, 

when then 

when then 

Now, 

































**Problem-12: Evaluate** 

**Solution: Let,** 

Put 

whenthen 

whenthen 

Now 



Again let 



whenthen 

whenthen 















**General Properties of Definite Integrals:** The general properties are,

1. 
2. 
3. 
4. 
5. 
6. 

**Question-01:** Prove that .

**Proof:** Let  and 





Again, 



From (i) and (ii) we have

 **(Thus proved)**

**Question-02:** Prove that .

**Proof:** Let 





Again, 



From (i) and (ii) we have

 **(Thus proved)**

**Question-03:** Prove that , when .

**Proof:** Let 





Again, 





From (i) and (ii) we have

 **(Thus proved)**

**Question-04:** Prove that .

**Proof:** Let , then 

whenthen 

whenthen 

Now 





 **(Thus proved)**

**Question-05:** Prove that  if .

**Proof:** Here, 



Now 

Let , then 

whenthen 

whenthen 









Similarly we have,



From (i) we have,





 **(Thus proved)**

**Question-06:** Prove that 

**Proof:** Here, 



Now 

Let , then 

whenthen 

whenthen 







From (i) we have,



If  then (ii) reduces as





Again, if  then (ii) reduces as





 **(Thus proved)**

**Question-07:** Prove that 

**Proof:** Here, 



Now 

Let , then 

whenthen 

whenthen 







From (i) we have,



If  i.e. is an even function, then (ii) reduces as



.

Again, if  i.e. is an odd function, then (ii) reduces as



.

 **(Thus proved)**

**Problem-01: Evaluate** 

**Solution: Let,** 







Now 











**Problem-02: Evaluate** 

**Solution: Let,** 





Now 











**Problem-03: Evaluate** 

**Solution: Let,** 





Now 























**Problem-04: Evaluate** 

**Solution: Let,** 





Now 





put

whenthen 

whenthen 



















**Problem-05: Evaluate** 

**Solution: Let,** 





Now 





























**Problem-06: Evaluate** 

**Solution: Let,** 

put

whenthen 

whenthen 















Now 











**Problem-07: Evaluate** 

**Solution: Let,** 





Now 











 … … … (1)

where,

put

whenthen 

whenthen 









From (1) we get







**Problem-08: Evaluate** 

**Solution: Let,** 





Now 









